SIMPLE RANDOM ASSIGNMENT – A PRIORI PROBABILITIES

METHOD 1 – EQUAL PROBABILITIES

EXPERIMENTAL UNITS

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

ASSIGNMENT MECHANISM

1 2 3 4

If 1 then 1 is assigned to T1 $p = 1/4 = .25$

If 2 then 1 is assigned to T2 $p = 1/4 = .25$

If 3 then 1 is assigned to T3 $p = 1/4 = .25$

If 4 then 1 is assigned to T4 $p = 1/4 = .25$

One draw is made for each experimental unit

The number of possible assignments of 20 experimental units (N objects) to 4 treatments (t boxes) is:

$$NPA = 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 1099511627776.$$

That is, $NPA = t^N = 4^{20} = 1099511627776$.

SIMPLE RANDOM ASSIGNMENT – A PRIORI PROBABILITIES

METHOD 2 – UNEQUAL PROBABILITIES

EXPERIMENTAL UNITS

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

ASSIGNMENT MECHANISM

1 2 3 4 5

If 1 then 1 is assigned to T1 $p_1 = 1/5 = .20$

If 2 then 1 is assigned to T2 $p_2 = 1/5 = .20$

If 3 then 1 is assigned to T3 $p_3 = 1/5 = .20$

If 4 then 1 is assigned to
If 5 then 1 is assigned to T4 $p_4 = 2/5 = .40$

One draw is made for each experimental unit

The number of possible assignments is identical to *Method 1*: $NPA = t^N = 4^{20} = 1099511627776$. However, in *Method 2* some assignments have a greater probability of being chosen (see explanation in the two treatments example).

SIMPLE RANDOM ASSIGNMENT – FORCED SIZES

METHOD 3 – EQUAL SIZES

EXPERIMENTAL UNITS

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

ASSIGNMENT MECHANISM

N = 20 *t = 4*

Step 1 – Draw of 5 balls from 20

This is *one* of the 15504 possible assignments

$$C_n^N = \frac{N!}{n!(N-n)!} \qquad C_5^{20} = \frac{20!}{5!(20-5)!} = 15504$$

Step 2 – Draw of 5 balls from 15

This is *one* of the 3003 possible assignments

$$C_5^{15} = \frac{15!}{5!(15-5)!} = 3003$$

Step 3 – Draw of 5 balls from 10

This is *one* of the 252 possible assignments

$$C_5^{10} = \frac{10!}{5!(10-5)!} = 252$$

Step 4 – No Draw

This is the *only* possible assignment

$$C_5^5 = \frac{5!}{5!(5-5)!} = 1$$

The number of possible assignments of 20 experimental units (*N* objects) to 4 treatments (*t* boxes), with an equal number of experimental units (*n* = 5) per treatment, is:

$$NPA = C_n^N \times C_n^{N-n} \times C_n^{N-2n} \times C_n^{N-3n} = C_5^{20} \times C_5^{15} \times C_5^{10} \times C_5^5 = 15504 \times 3003 \times 252 \times 1 = 11732745024$$

A more convenient formula to calculate this number is:

$$NPA = \frac{N!}{(n!)^t} = \frac{20!}{(5!)^4} = 11732745024$$

SIMPLE RANDOM ASSIGNMENT – FORCED SIZES

METHOD 4 – UNEQUAL SIZES

EXPERIMENTAL UNITS

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

ASSIGNMENT MECHANISM ***N* = 20** ***t* = 4**

Step 1 – Draw of 7 balls from 20

This is *one* of the 77520 possible assignments

$$C_n^N = \frac{N!}{n!(N-n)!} \qquad C_7^{20} = \frac{20!}{7!(20-7)!} = 77520$$

Step 2 – Draw of 4 balls from 13

This is *one* of the 715 possible assignments

$$C_4^{13} = \frac{13!}{4!(13-4)!} = 715$$

Step 3 – Draw of 5 balls from 9

This is *one* of the 126 possible assignments

$$C_5^9 = \frac{9!}{5!(9-5)!} = 126$$

Step 4 – No Draw

This is the *only* possible assignment

$$C_4^4 = \frac{4!}{4!(4-4)!} = 1$$

The number of possible assignments of 20 experimental units (*N* objects) to 4 treatments (*t* boxes), with an unequal number of experimental units (n_1, n_2, n_3, n_4) per treatment, is:

$$NPA = C_{n_1}^{N} \times C_{n_2}^{N-n_1} \times C_{n_3}^{N-n_1-n_2} \times C_{n_4}^{N-n_1-n_2-n_3} = C_7^{20} \times C_4^{13} \times C_5^{9} \times C_4^{4} = 77520 \times 715 \times 126 \times 1 = 6983776800$$

A more convenient formula to calculate this number is:

$$NPA = \frac{N!}{(n_1!)(n_2!)(n_3!)(n_4!)} = \frac{20!}{(7!)(4!)(5!)(4!)} = 6983776800$$

REFERENCES

Alferes, V. R. (2012). Methods of randomization in experimental design. Thousand Oaks, CA: Sage [See pp. 23-27].

Alferes, V. R. (2012). SCRAED: *Simple and complex random assignment in experimental design* [SPSS Syntax Files – Version 2.0]. Available at www.fpce.uc.pt/niips/randmethods

APPENDIX 1

SIMPLE RANDOM ASSIGNMENT – COUNTING NUMBER OF POSSIBLE ASSIGNMENTS

***N* = 20** ***t* = 4**

Number of Partitions of 20 into 4 parts = 108

(64 Nonzero Partitions + 44 Partitions Including Zero Parts)

Partition			Number of Experimental Units per Partition (n_i)				Random Assignments per Partition		
No.	Type		Partition 1 [T_1]	Partition 2 [T_2]	Partition 3 [T_3]	Partition 4 [T_4]	Number of Possible Assignments (*Non Ordered* Treatments)	Possible Arrangements of Treatments	Number of Possible Assignments (*Ordered* Treatments)
		N	n_1	n_2	n_3	n_4	$NPA_{nord} = \frac{N!}{(n_1!)(n_2!)(n_3!)(n_4!)}$	A See Appendix 2 (Examples)	$NPA_{ord} = NPA_{nord} \times A$
1	Zero Parts	20	20	0	0	0	1	4	4
2	Zero Parts	20	19	1	0	0	20	12	240
3	Zero Parts	20	18	2	0	0	190	12	2280
4	Zero Parts	20	18	1	1	0	380	12	4560
5	Zero Parts	20	17	3	0	0	1140	12	13680
6	Zero Parts	20	17	2	1	0	3420	24	82080
7	No Zero Parts	20	17	1	1	1	6840	4	27360
8	Zero Parts	20	16	4	0	0	4845	12	58140
9	Zero Parts	20	16	3	1	0	19380	24	465120
10	No Zero Parts	20	16	2	1	1	58140	12	697680
11	Zero Parts	20	16	2	2	0	29070	12	348840
12	Zero Parts	20	15	5	0	0	15504	12	186048
13	Zero Parts	20	15	4	1	0	77520	24	1860480
14	No Zero Parts	20	15	3	1	1	310080	12	3720960
15	Zero Parts	20	15	3	2	0	155040	24	3720960
16	No Zero Parts	20	15	2	2	1	465120	12	5581440
17	Zero Parts	20	14	6	0	0	38760	12	465120
18	Zero Parts	20	14	5	1	0	232560	24	5581440
19	No Zero Parts	20	14	4	1	1	1162800	12	13953600
20	Zero Parts	20	14	4	2	0	581400	24	13953600
21	No Zero Parts	20	14	3	2	1	2325600	24	55814400
22	Zero Parts	20	14	3	3	0	775200	12	9302400
23	No Zero Parts	20	14	2	2	2	3488400	4	13953600
24	Zero Parts	20	13	7	0	0	77520	12	930240
25	Zero Parts	20	13	6	1	0	542640	24	13023360
26	No Zero Parts	20	13	5	1	1	3255840	12	39070080
27	Zero Parts	20	13	5	2	0	1627920	24	39070080
28	No Zero Parts	20	13	4	2	1	8139600	24	195350400
29	Zero Parts	20	13	4	3	0	2713200	24	65116800
30	No Zero Parts	20	13	3	2	2	16279200	12	195350400
31	No Zero Parts	20	13	3	3	1	10852800	12	130233600
32	Zero Parts	20	12	8	0	0	125970	12	1511640
33	Zero Parts	20	12	7	1	0	1007760	24	24186240
34	No Zero Parts	20	12	6	1	1	7054320	12	84651840
35	Zero Parts	20	12	6	2	0	3527160	24	84651840
36	No Zero Parts	20	12	5	2	1	21162960	24	507911040
37	Zero Parts	20	12	5	3	0	7054320	24	169303680
38	No Zero Parts	20	12	4	2	2	52907400	12	634888800
39	No Zero Parts	20	12	4	3	1	35271600	24	846518400
40	Zero Parts	20	12	4	4	0	8817900	12	105814800
41	No Zero Parts	20	12	3	3	2	70543200	12	846518400
42	Zero Parts	20	11	9	0	0	167960	12	2015520
43	Zero Parts	20	11	8	1	0	1511640	24	36279360

Partition			Number of Experimental Units per Partition (n_i)				Random Assignments per Partition		
No.	Type	N	Partition 1 [T_1] n_1	Partition 2 [T_2] n_2	Partition 3 [T_3] n_3	Partition 4 [T_4] n_4	Number of Possible Assignments (*Non Ordered* Treatments) $NPA_{nord} = \frac{N!}{(n_1!)(n_2!)(n_3!)(n_4!)}$	Possible Arrangements of Treatments A See Appendix 2 (Examples)	Number of Possible Assignments (*Ordered* Treatments) $NPA_{ord} = NPA_{nord} \times A$
44	No Zero Parts	20	11	7	1	1	12093120	12	145117440
45	Zero Parts	20	11	7	2	0	6046560	24	145117440
46	No Zero Parts	20	11	6	2	1	42325920	24	1015822080
47	Zero Parts	20	11	6	3	0	14108640	24	338607360
48	No Zero Parts	20	11	5	2	2	126977760	12	1523733120
49	No Zero Parts	20	11	5	3	1	84651840	24	2031644160
50	Zero Parts	20	11	5	4	0	21162960	24	507911040
51	No Zero Parts	20	11	4	3	2	211629600	24	5079110400
52	No Zero Parts	20	11	4	4	1	105814800	12	1269777600
53	No Zero Parts	20	11	3	3	3	282172800	4	1128691200
54	Zero Parts	20	10	10	0	0	184756	6	1108536
55	Zero Parts	20	10	9	1	0	1847560	24	44341440
56	No Zero Parts	20	10	8	1	1	16628040	12	199536480
57	Zero Parts	20	10	8	2	0	8314020	24	199536480
58	No Zero Parts	20	10	7	2	1	66512160	24	1596291840
59	Zero Parts	20	10	7	3	0	22170720	24	532097280
60	No Zero Parts	20	10	6	2	2	232792560	12	2793510720
61	No Zero Parts	20	10	6	3	1	155195040	24	3724680960
62	Zero Parts	20	10	6	4	0	38798760	24	931170240
63	No Zero Parts	20	10	5	3	2	465585120	24	11174042880
64	No Zero Parts	20	10	5	4	1	232792560	24	5587021440
65	Zero Parts	20	10	5	5	0	46558512	12	558702144
66	No Zero Parts	20	10	4	3	3	775975200	12	9311702400
67	No Zero Parts	20	10	4	4	2	581981400	12	6983776800
68	No Zero Parts	20	9	9	1	1	18475600	6	110853600
69	Zero Parts	20	9	9	2	0	9237800	12	110853600
70	No Zero Parts	20	9	8	2	1	83140200	24	1995364800
71	Zero Parts	20	9	8	3	0	27713400	24	665121600
72	No Zero Parts	20	9	7	2	2	332560800	12	3990729600
73	No Zero Parts	20	9	7	3	1	221707200	24	5320972800
74	Zero Parts	20	9	7	4	0	55426800	24	1330243200
75	No Zero Parts	20	9	6	3	2	775975200	24	18623404800
76	No Zero Parts	20	9	6	4	1	387987600	24	9311702400
77	Zero Parts	20	9	6	5	0	77597520	24	1862340480
78	No Zero Parts	20	9	5	3	3	1551950400	12	18623404800
79	No Zero Parts	20	9	5	4	2	1163962800	24	27935107200
80	No Zero Parts	20	9	5	5	1	465585120	12	5587021440
81	No Zero Parts	20	9	4	4	3	1939938000	12	23279256000
82	No Zero Parts	20	8	8	2	2	374130900	6	2244785400
83	No Zero Parts	20	8	8	3	1	249420600	12	2993047200
84	Zero Parts	20	8	8	4	0	62355150	12	748261800
85	No Zero Parts	20	8	7	3	2	997682400	24	23944377600
86	No Zero Parts	20	8	7	4	1	498841200	24	11972188800
87	Zero Parts	20	8	7	5	0	99768240	24	2394437760
88	No Zero Parts	20	8	6	3	3	2327925600	12	27935107200
89	No Zero Parts	20	8	6	4	2	1745944200	24	41902660800
90	No Zero Parts	20	8	6	5	1	698377680	24	16761064320
91	Zero Parts	20	8	6	6	0	116396280	12	1396755360
92	No Zero Parts	20	8	5	4	3	3491888400	24	83805321600
93	No Zero Parts	20	8	5	5	2	2095133040	12	25141596480
94	No Zero Parts	20	8	4	4	4	4364860500	4	17459442000
95	No Zero Parts	20	7	7	3	3	2660486400	6	15962918400

Partition			Number of Experimental Units per Partition (n_i)				Random Assignments per Partition		
No.	Type		Partition 1 [T_1]	Partition 2 [T_2]	Partition 3 [T_3]	Partition 4 [T_4]	Number of Possible Assignments (*Non Ordered* Treatments)	Possible Arrangements of Treatments	Number of Possible Assignments (*Ordered* Treatments)
		N	n_1	n_2	n_3	n_4	$NPA_{nord} = \frac{N!}{(n_1!)(n_2!)(n_3!)(n_4!)}$	A See Appendix 2 (Examples)	$NPA_{ord} = NPA_{nord} \times A$
96	No Zero Parts	20	7	7	4	2	1995364800	12	23944377600
97	No Zero Parts	20	7	7	5	1	798145920	12	9577751040
98	Zero Parts	20	7	7	6	0	133024320	12	1596291840
99	No Zero Parts	20	7	6	4	3	4655851200	24	111740428800
100	No Zero Parts	20	7	6	5	2	2793510720	24	67044257280
101	No Zero Parts	20	7	6	6	1	931170240	12	11174042880
102	No Zero Parts	20	7	5	4	4	6983776800	12	83805321600
103	No Zero Parts	20	7	5	5	3	5587021440	12	67044257280
104	No Zero Parts	20	6	6	4	4	8147739600	6	48886437600
105	No Zero Parts	20	6	6	5	3	6518191680	12	78218300160
106	No Zero Parts	20	6	6	6	2	3259095840	4	13036383360
107	No Zero Parts	20	6	5	5	4	9777287520	12	117327450240
108	No Zero Parts	20	5	5	5	5	11732745024	1	11732745024
								Total Number of Possible Assignments = $\sum NPA_{ord}$ =	1099511627776

METHODS 1 and 2

Number of Possible Assignments: All Partitions and All Arrangements of Treatments Within Partition

$\sum NPA_{ord} = 1099511627776$

$t^N = 4^{20} = 1099511627776$

METHOD 3 ($T_1 = 5$; $T_2 = 5$; $T_3 = 5$; $T_4 = 5$)

Number of Possible Assignments: Partition 108

$$NPA_{nord} = \frac{N!}{(n_1!)(n_2!)(n_3!)(n_4!)} = \frac{N!}{(n!)^t} = \frac{20!}{(5!)(5!)(5!)(5!)} = \frac{20!}{(5!)^4} = 11732745024$$

If you use **Method 1**, instead of **Method 3**, the probability of getting a random assignment with equal sizes per treatment is:

p **=** 11732745024/1099511627776 **= 0.0107**

METHOD 4 ($T_1 = 7$; $T_2 = 4$; $T_3 = 5$; $T_4 = 4$)

Number of Possible Assignments: Partition 102/Arrangment 2

Possible Arrangements	*n* per Treatment				Number of Possible Assignments
	T_1	T_2	T_3	T_4	
1	7	5	4	4	$NPA_{nord} = \frac{N!}{(n_1!)(n_2!)(n_3!)(n_4!)} = \frac{20!}{(7!)(5!)(4!)(4!)} = 6983776800$
2	7	4	5	4	$NPA_{nord} = \frac{N!}{(n_1!)(n_2!)(n_3!)(n_4!)} = \frac{20!}{(7!)(4!)(5!)(4!)} = 6983776800$
3	7	4	4	5	[...]
4	5	7	4	4	[...]
5	5	4	7	4	[...]
6	5	4	4	7	[...]
7	4	7	5	4	[...]
8	4	7	4	5	[...]
9	4	5	7	4	[...]
10	4	5	4	7	[...]
11	4	4	7	5	[...]
12	4	4	5	7	$NPA_{nord} = \frac{N!}{(n_1!)(n_2!)(n_3!)(n_4!)} = \frac{20!}{(4!)(4!)(5!)(7!)} = 6983776800$

If you use **Method 1**, instead of **Method 4**, the probability of getting a random assignment with treatment levels sizes $n_1 = 7$, $n_2 = 4$, $n_3 = 5$, and $n_4 = 4$ is:

p **=** 6983776800/1099511627776 **= 0.0064**

APPENDIX 2

EXAMPLES OF POSSIBLE ARRANGEMENTS OF TREATMENTS WITHIN PARTITIONS

Partitions with *four* different "*n* parts"

Example: Partition 6

Possible Arrangements	*n* per Treatment			
	T_1	T_2	T_3	T_4
1	17	2	1	0
2	17	2	0	1
3	17	1	2	0
4	17	1	0	2
5	17	0	2	1
6	17	0	1	2
7	2	17	1	0
8	2	17	0	1
9	2	1	17	0
10	2	1	0	17
11	2	0	17	1
12	2	0	1	17
13	1	17	2	0
14	1	17	0	2
15	1	2	17	0
16	1	2	0	17
17	1	0	17	2
18	1	0	2	17
19	0	17	2	1
20	0	17	1	2
21	0	2	17	1
22	0	2	1	17
23	0	1	17	2
24	0	1	2	17

Partitions with *two* different "*n* parts"

Example: Partition 1

Possible Arrangements	*n* per Treatment			
	T_1	T_2	T_3	T_4
1	20	0	0	0
2	0	20	0	0
3	0	0	20	0
4	0	0	0	20

Example: Partition 54

Possible Arrangements	*n* per Treatment			
	T_1	T_2	T_3	T_4
1	10	10	0	0
2	10	0	10	0
3	10	0	0	10
4	0	10	10	0
5	0	10	0	10
6	0	0	10	10

Partitions with *three* different "*n* parts"

Example: Partition 2

Possible Arrangements	*n* per Treatment			
	T_1	T_2	T_3	T_4
1	19	1	0	0
2	19	0	1	0
3	19	0	0	1
4	1	19	0	0
5	1	0	19	0
6	1	0	0	19
7	0	19	1	0
8	0	19	0	1
9	0	1	19	0
10	0	1	0	19
11	0	0	19	1
12	0	0	1	19

Partitions with *no* different "*n* parts"

Example: Partition 108

Possible Arrangements	*n* per Treatment			
	T_1	T_2	T_3	T_4
1	5	5	5	5

Alferes, V. R. (2012). *Methods of randomization in experimental design*. Thousand Oaks, CA: Sage. doi: http://dx.doi.org/10.4135/9781452270012

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